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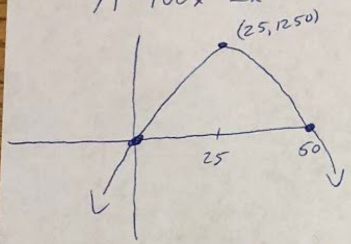
Math 164
Quantitative Reasoning Paper

For this optimization scenario, a homeowner wanted to fence in their rectangular garden using the river in their backyard as one of the sides. They only have a total of 100 feet of fencing to use to complete their project. The homeowners reached out to find the dimensions that would yield the largest possible garden and I believe I have found the solution.

I first started going about this problem by identifying the independent and dependent variables. The dependent variable relies on the independent variable to produce an outcome. The independent variable in this case is the lengths of the sides of the garden while the dependent variable is the area of the garden. The dependent variable lays on the y axis and the independent variable lays on the y axis. The dependent variable is the area of the garden because it depends on the independent variable (the lengths of the sides of the garden) to produce an outcome which is called the area. The function I derived to represent this scenario is <u>Area=100x-2x^2.</u> I came up with this function by allowing the variable y=the length of the side parallel to the river and x=the length of the 2 other sides of the fence perpendicular to the fence. So the perimeter of the fencing is <u>y+2x=100</u> because y is the length of the side parallel to the fence, there is only one side and x represents the sides perpendicular to the river, there should be two sides, which is where the 2x comes from. Next, we have to solve the perimeter function for y. Once we do that we should get <u>y=100-2x</u> and now we are ready to substitute that equation into the formula for the area of a rectangle, <u>A=lw.</u> We can manipulate this formula to better suit this problem by making it <u>A=xy</u>, where x and y now represent the sides of the fenced in garden. Earlier we found that <u>y=100-2x</u> so we can substitute that equation into the area equation (A=xy) for y. Now, the equation for the area is <u>A=100x-2x^2</u> and we can recognize this is a parabola with vertex (25,1250), and with zeros at (0,0) and (50,0). To find the vertex of the equation we can factor a -2 out of <u>A=100x-2x^2</u> and complete the square by dividing 100 by 2 and squaring it, then adding and subtracting that same number into the new equation, giving us A=-2((x^2-50x+625)-625). Then you must factor out the x^2-50x+625 giving you (x-25)^2. Then distribute the -2 to the -625 outside of the parenthesis giving the quadratic in graphing form, A=-2(x-25)^2+1250. This is how we can see the vertex, (x-25)^2 will give us the x coordinate of the vertex. Since it is inside of the parenthesis, the graph is shifted to the right 25 units, making the x value= 25. To find the y value of the vertex, plug 25 in for x to A=100x-2x^2. This gives us the vertex (25,1250). This graph crosses the x axis at 2 places. To find these points, set the equation A=100x-2x^2 equal to zero. We can factor this and get the zeros of this function to be (0,0) and (50,0). The vertex is important because it is a maximum. It is a maximum because there is a negative coefficient in its graphing form, A=-2(x-25)^2+1250. The y value of the coefficient, 1250, is the maximum area the garden can have given that only 100 feet of fence is given to make the fence.

$A = 100x - 2x^2$



So, the max. area is 1250. This occurs when $x=25$.

- Plug $x=25$ into the perimeter equation

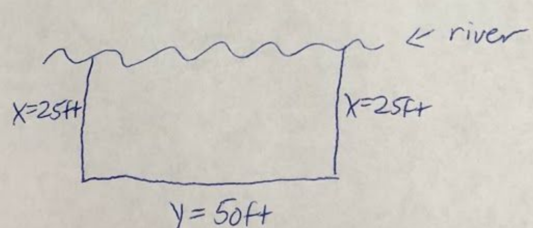
$y = 100 - 2x$

$y = 100 - 2(25)$

$y = 50ft$

Domain: $[0, 50]$
Range: $[0, 1250]$ > Since length can't be negative

Since y = length of the side parallel to the river, This side's length is 50ft long. And since x = length of the other two sides, these sides are both 25ft long.



We know that since the vertex is at (25,1250), the maximum area of the garden occurs when $x=25$. Since we know x, the two side lengths perpendicular to the river, are 25 feet long. We can find the length of the side parallel to the river by plugging $x=25$ into our perimeter equation $y=100-2x$ to get that the side parallel to the river is 50 feet long.

The domain of the function is theoretically all real numbers, but since we cannot have negative length, the actual domain is [0,50]. The same thing goes for the range, we cannot have negative length so the range is [0,1250]. The domain is all the x values that satisfy this equation and the range is all of the y values that satisfy the equation. For example, since the range of this function is [0,1250], this means the area of the garden can be no greater than 1250 ft^2. My solution is reasonable because the function I derived $\underline{A=100x-2x^2}$ gives the same solution to the maximum area as if I were to multiply the length of the garden by the width and no lengths are negative or zero.

Since I chose y= the length of the side parallel to the river, by plugging $x=25$ (which was the x coordinate of our vertex) to our perimeter equation, we found the length of the side parallel to the river to be 50 feet and the two sides perpendicular to the river to be each 25 feet long. This also makes sense because if you were to multiple the length of the garden by the width, you would get the maximum area to be 1250 ft^2. Just like the equation we derived said it was going to be.