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Math 121

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Exponential Growth Project

If you are thinking about investing into an account, it is important to pay attention to the interest rates. It is very important to find an account with an annual interest rate of at least 3% per year. This is because if an interest rate is below 3%, the amount of interest you will earn will not exceed the amount of personal investments you make into the account. However, if you find an account with an interest rate of at least 3% or higher and invest money for a long period of time, such as 15 years, then the amount of interest you earn in the account will greatly exceed your own personal investments into the account.

Assume that you make a payment of $1,500 into an account quarterly with an annual interest rate of 4% for 46 years: from your 22nd birthday until your 68th birthday. The value of your account on your 68th birthday would be $803,199.69. The total amount of money you would personally invest into the account is $277,500 so the total amount of interest you would earn on the account is $525,699.69. However, if you were to double your quarterly payments to $3,000, the value of your account on your 68th birthday would be $1,606,399.39. Notice that if you were to double your payments then the final value of your account after the 46 years would also double.

Suppose that you make the same payment of $1,500 into an account quarterly with an annual interest rate of 7% over the same period of time. On your 68th birthday, the balance of your account would be $2,072,791.62. At this point in time, you still would have invested $277,500, but because of the higher interest rate, the final value of the account will be $1,269,591.93 more than it was in the account with the 4% interest rate. Doubling your quarterly payments would result in an even higher account balance on your 68th birthday: $4,145,583.24. This would result in a final account balance of $2,539,183.86 more than the final balance in the account with the 4% interest when your personal investments are $3,000 every quarter. You can see that doubling your payments still results in a doubled final value of the account.

Now suppose that you put the same quarterly payment of $1,500 into an account with an annual interest rate of 10%. After 46 years, on your 68th birthday, your account will have a value of $5,864,889.80. Your total investments would still be $277,500. The total amount of interest you earn on the account would be $5,587,389.90. Once again, if you were to double your quarterly payments into your account to $3,000, the final value in the account would be $11,729,779.60, which is double the value of the account when you were investing $1,500 quarterly rather than $3,000 quarterly.

If you put $1,500 into an account quarterly with an interest rate of 8%, your account will be worth $2,906,774.47 rather than the $803,199.69 you would have in the 4% account. Once again, if you were to double your interest rate from 7% to 14%, your account value would be $25,714,599.98 compared to $2,072,791.62. Finally, if you double your rate from 10% to 20%, the value of your account on your 68th birthday would be $261,986,351.80. The value of your accounts when the interest rate is doubled is significantly higher. It would be most beneficial to find an account with the highest interest rate in order to have more money by your 68th birthday.

Suppose you start making your quarterly payments of $1,500 at age 22, but are only able to continue making them until you are 42 years old. In this case, by your 68th birthday, the money that sat in the account from the time you were 42 until you turned 68 would still have accrued interest, even though you had not been making payments into the account. In this case, the total value of your account on your 68th birthday would be $528,281.71 if the interest rate were 4%. In the same situation, the total value of your account on your 68th birthday would be $1,630,138.64 if the account’s interest rate were 7%. Again, under the same circumstances, if the account’s interest rate were 10%, the account’s total value on your 68th birthday would be $5,124,421.66.

Let’s say that you still want to begin investing at age 22, but you want to stop making payments at age 45 and just let whatever is in the account continue to accrue interest until your 68th birthday. However, you still want the total account balance on your 68th birthday to closely match what it would have been if you had been making quarterly $1,500 payments from your 22nd all the way through your 68th birthday, like we have previously discussed. You know that you must increase the amounts of your quarterly payments into the account in order to make this work, but by how much? In order to closely match the ending balance of the 4% account with the quarterly $1,500 payments starting at age 22, you would have to make quarterly payments of $3,434.73 from the time you were 22 until you were 45. In order to closely match the ending balance of the 7% account with the quarterly $1,500 payments starting at age 22, from the time you were 22 until your 45th birthday, you would have to start making $6,974.38 payments every quarter. And to closely match the ending balance of the 10% account with the quarterly $1,500 payments starting at age 22, from the time you were 22 until your 45th birthday you would have to begin investing $13,982.92 every quarter.

Now let’s say that you do not want to begin investing money into an account on your 22nd birthday and you want to wait until your 45th birthday instead. If you wait until you are 45, the money you are investing into the account will only be there for 23 years. If you invest $1,500 into an account with an annual 4% interest rate beginning on your 45th birthday, by the time you are 68 years old you will have $230,708.55. Additionally, you will have earned $91,208.55 in interest. However, if you would like to have the same ending balance in your account as if you were investing from 22 to 68, then you would have to make quarterly payments of approximately $5,222.17.

Similarly, if you decided to invest at age 45 into an account with an annual interest rate of 7%, your account on your 68th birthday would have a value of $350,598.55 and you would earn $211,098.55 in interest. If you would like your ending balance to closely match what it was when you began investing at 22, $2,072,791.62, then you would have to make quarterly payments of about $8,868.22. Your end value in this instance would be $2,072,790.06.

Finally, if you were to invest into an account with an annual interest rate of 10% beginning on the day you turn 45, your account would have a value of $549,715.84 on your 68th birthday. You would also earn about $410,215.84 in interest. On the other hand, if you would still like your end balance to be the same as it would be if you were to begin investing at age 22, then your quarterly payments would have to be approximately $16,003.42.

Let’s assume again that you would like to wait until you turn 45 years old to begin investing. However, you would like to make up for the time you missed while still making the same quarterly payment of $1,500; you would need to find an account with a very high interest rate so that your account value when you turn 68 is $2,072,791.62: the value of your account with a 7% annual interest rate. In order to closely match this ending balance, you would have to find an account with an interest rate of roughly 18.055%.

As you can see, the earlier you can begin making investments into a savings account, the better. The longer an investment can sit in an account, the more interest it will accrue. As interest begins to build up, the interest earned will start earning interest, too. That is why starting accounts at an early age proves to be so beneficial. Otherwise, in order to end up with the same amount of money by a certain point in time, your own personal investments will have to increase and the amount of interest you earn will decrease.