

Colleen and Bill have three different options to pay for the house they purchased. Because the seller has a second mortgage of \$50,000 for 5 years, he has offered to have Colleen and Bill pay back that second mortgage as payment for the house. The seller gives Colleen and Bill these three options: 1. simple interest at 12% annually, 2. 11.5% interest compounded quarterly, and 3. 11.25% interest compounded monthly.

Bill and Colleen need to mathematically figure out how much interest they will pay for each loan option using the simple interest formula and the compound interest formula. The simple interest formula is  $interest = principal * interest\ rate$ . This is for one year. If simple interest needs to be found for multiple years, multiply the interest found by the number of years. In order to find the total amount they will spend, they have to add the interest found in the formula to the principal. For Bill and Colleen's first option,  $interest = 50,000 * 0.12 = 6,000$ . If the interest is 6,000 for each year, then for 5 years interest will be  $6,000 * 5 = 30,000$ . For the simple interest Colleen and Bill will be paying \$30,000 on top of their principal of \$50,000, for a combined total of \$80,000.

When calculating each year individually the simple interest formula can be written as  $total = principal + principal * rate$ . Using algebra, the principal can be factored out so that the equation would be  $total = principal(1 + rate)$ . When interest is compounded more than once per year the rate is for the total year, not each time the interest is compounded, so the formula becomes  $total = principal(1 + (rate/number\ of\ times\ compounded\ per\ year))$ . Number of times compounded per year will be written as  $n$  in all following equations. Since the equation so far only calculates interest once, the  $(1 + (rate/n))$  needs to be multiplied by the number of times the interest needs to be calculated.  $(1 + (rate/n))$  could be written out and multiplied as many times as needed, or raised to a power. The power is equivalent to  $n * years$ , since that is the number of times interest is being calculated in total. For example, if the interest is calculated 4 times per year over 10 years, it would be raised to the power of 40 since interest is being calculated that many times over the life of the loan. The appropriate formula to figure out compounding interest for the rest of Colleen and Bill's options is therefore,  $total = principal(1 + (rate/n))^{n*years}$ .

For the second option Colleen and Bill have an interest rate of 0.115 compounded quarterly, so 4 times per year. By using the formula created above,  $total = principal(1 + (rate/n))^{n*years}$ , then plugging in the given information I got  $total = 50,000(1 + (0.115/4)^{4*5} = 88,138.77$ . Since that is \$88,138.77 over the life of the loan \$50,000 needs to be subtracted from it to find only the interest they will have to pay. Based on this subtraction,  $88,138.77 - 50,000 = 38,138.77$  Colleen and Bill will be paying \$38,138.77 in interest for the second option.

For the third and final option Bill and Colleen have an interest rate of 0.1125 compounded monthly, or 12 times per year. By using the formula created above,  $total = principal(1 + (rate/n))^{n*years}$ , then plugging in the given information, like so  $total = 50,000(1 + (0.1125/12)^{12*5}$ , I found that the total will be \$87,523.09. Since that is the

total amount of money they will pay over the life of the loan, the principal needs to be subtracted from that,  $87,523.09 - 50,000 = 37,523.09$ . With the final option Bill and Colleen will be paying \$37,523.09 in interest.

Since  $\$30,000 < \$37,523.09 < \$38,138.77$ , Bill and Colleen should choose option one in order to pay the least amount. This solution is reasonable because the simple interest was only added 5 times, while the others were added 20 and 60 times respectively. The interest rates were half a percent and three quarters of a percent less, so it was not significant enough of a difference to make up for the additional times the interest was being calculated. I made the assumption that all the interest rates were fixed and not variable. Variable rates would change. Since money is only calculated to two decimal places, I made assumptions when rounding in my calculations.

Since the option of 12% simple interest accumulated the least in interest on the \$50,000 second mortgage, Colleen and Bill should choose that option to pay the least amount of money on their home purchase.